Puzzle
The mathematics behind Escher

Inside View
Imaginary numbers of engineering problems

Horoscope
See into your future

Francken Vrij

27.2 Imaginary

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Editorial
Next to our theme, our layoutweekend was also imaginary. We still managed to put together the edition you're reading right now during an amusing layout evening. The committee imagined an edition including a committee experience rubric, but we weren't able to realize it yet, since we were too busy during the first try and too drunk during the second one. Hopefully the third edition is a charm and you will be able to enjoy this new rubric about an adventure of the committee. We are still very pleased to present to you already the second edition of our year! Full of interesting physics, but next to that a lot of other reading material for you to enjoy.

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Edition 27.2

Chair's preface

By Csilla Tijssen

Dear Francken members, as we explore the fascinating world of applied physics, we often encounter concepts that may seem intangible or elusive, yet they play a crucial role in shaping our understanding of the natural world. Some of the most fundamental concepts in physics, such as atoms, particles, and forces, are entirely imaginary constructs that were developed to help us understand the world around us. Even the laws of physics themselves are based on mathematical models that are essentially imaginary, but which have proven to be incredibly accurate at predicting the behavior of the physical universe.

This really shows how important it is to have a strong imagination as a physicist. Without people imagining new things, no new advances can be made within physics, the only way that we can keep moving forward is by stepping into imagined realms which either turn out to be correct or urge us to use more of our imagination and create another idea. One such thing that seemed to be impossible at first, but which people dared to imagine and strive to create was a being that can give an answer to anything. First we had Google, where you could find your answers eventually if you kept looking long enough. Now we have ChatGPT which makes it a lot easier to find the answers to your questions (even though sometimes not entirely correctly) and can even create entire pieces of text. Who could've imagined a century ago that we'd actually be conversing with machines today?

Finally I would like to end this piece with something to think about: what if this entire piece was actually not written by me, but created by ChatGPT answering a few questions of mine? Would you mind, would you think I cheated or just used the tools which I had to my disposal?

News of the Association

By Filippo Carretta

Dear Francken members, the moment has come for yet another insider's view, in order for you to have a look at all the events that have happened till now. I can tell you, though, that there are a lot of events to recap. However, here I will only be summarizing a few of these wonderful events. In addition to this, my fellow board member Christian asked me whether I had hidden a 'word' in my section last edition, which I did not. For this edition, though, I have decided to hide a word for you to find in my short section. Best of luck :)

Halloween Party

Intercie, the new Francken committee, organized its first event of the year which was a Halloween party held in the Francken room. Here Francken members came with their scariest costumes to induce fear in as many of their friends as possible. I, unfortunately, missed the event as I might have been too scared to even attend (maybe).

Ledenweekend

The first member's weekend of the year happened at de Strubben where 44 Francken members came together to drink the klok to beat the clock. Here members became closer together and shared experiences/secrets with each other. This is not to mention the epic cantus where members showed how singing gets better as they drank more. All in all the event brought members closer and proved as a solid first event for the Wiecksie.

Belsimpel in-house day

Belsimpel, located in the centre of Groningen, became the location for a great in-house day where members got a real hands-on experience of what it feels like working at Belsimpel. This was done through the challenging case study where our coding knowledge came to practical use. In addition to this, I can't underestimate that we also got free pizza at this event.

Tour de Francken

The great Tour and this year's tour went down in history as being the first tour won by a female member in Francken, Hester Braaksma. Here only the elite members of Francken showed their greatest skills in drinking faster than you can say "The theme of Francken Vrij 27.2 is Imaginary".

Christmas dinner

A wholesome Christmas dinner was held in the usual NB corridor and organized by the hardworking Fraccie. Francken members had a blast as they dressed up in their ugliest Christmas sweaters to show off who has the best/worst fashion tastes. The dinner proved as a unique opportunity for Francken members to say goodbye to each other before their two-week Christmas break.

Buixie announcement

London, Oxford, and Edinbrugh…… these are the locations of this year's Buixie as we head to the UK to visit their top-end universities and their research stations. From Italy/Switzerland to France to now England, Buixie is slowly moving up the map and

closer to the north pole. The Buixie board had a great announcement where they had a fun game to play where you could win shots if you got the right answer.

Everything but a cup borrel

Just as the name suggests this borrel showed how imaginative each of the Francken members is as they all came with different cups. Here I saw incredibly unique cups from a shoe, to a traffic cone to a really big cup.

Jamsession

An event was held with our astronomy buddies Sirius A where we sang and jammed together in this epic session. It could have Word been observed as a competition between who has the most talented singers, however, for this session, it was a peaceful unique experience where members from both associations enjoyed each other's company. We were still the better singers and musicians :)

Applied Physics dinner

The Applied Physics dinner was held in the NB cafeteria where professors shared a wonderful buffet with Applied Physics students. This year the dinner numbered seventy people and they all enjoyed a chat and a meal with each other as we all learned more about what happens in the field of research in the halls of building NB.

Thales Excursion

Yet another excursion organized by our External Sjoerd. Similar to our Nedap excursion we went in vans to the company Thales where they design satellites for security purposes. Here we saw some unique science and designs being put to the test as they construct revolutionary satellites for security.

Imaginary numbers in some engineering problems

By Rodolfo Reyes-Báez

This note aims to motivate why the abstract concept of complex numbers is crucial in understanding the behavior of physical systems. To this end, some well-chosen examples of basic (yet fundamental) engineering systems are presented (Control Tutorials). From now on, the author of this note assumes the reader has a working knowledge of complex numbers algebra: addition, multiplications, complex conjugate, module, and argument; calculus and Laplace transform. Nevertheless, the technical part can be often omitted and just follow the analysis conceptually.

Complex numbers and complex variables.

The equation $x^2 + 1 = 0$ does not have a well-defined solution on the set of real numbers $\mathbb{R}$. This fact motivated mathematicians to invent another set of numbers in which the equation above has a well-defined solution. This set is the so-called set of complex numbers $\mathbb{C}$ which is defined as the set of numbers of the form $s = a + bi$, with $a,b \in \mathbb{R}$ and $i^2 = -1$. The letter i is the imaginary unit, sometimes denoted j in engineering to avoid misunderstanding the electrical current notation. The number a is the real part of z, denoted as $a = \mathrm{Re}\{s\}$, and b is the imaginary part of z denoted by $b = \mathrm{Im}\{s\}$. A *complex variable* is a variable that takes values on the set of complex number; note that a real variable is a complex variable with zero imaginary part. Similarly, a *complex function* $f\colon \mathbb{C} \to \mathbb{C}$ is a function that maps a complex number s to a complex number $f(s)$ (Nise, 2019).

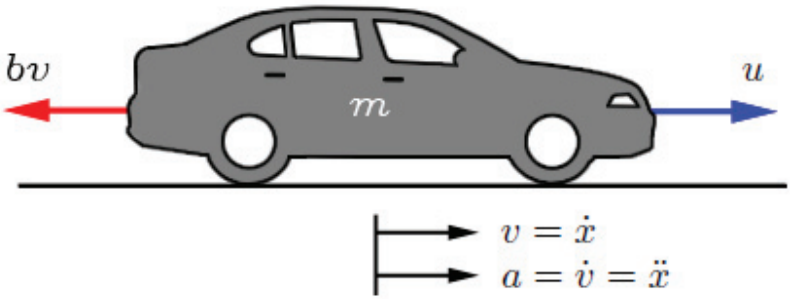

Figure 1: Car (Cruise Control: System Modeling), with position x, velocity y, and acceleration a. The constant b is the Coulomb friction coefficient and m the car's mass.

Complex numbers in a Car's cruise velocity analysis

Consider the problem of describing the velocity profile of a car along a (flat) path. To this end, one needs to compute the equation of motion that describes the time evolution of the car's velocity.

The free-body diagram in Figure 1. describes the direction in which the forces are acting on the car. For instance, u is a positive force associated with the motor and bu is the friction force between the tire and the road. Recall Newton's second law:

$$ma = \sum_{k=1}^{n} F_k$$

Then, the equation of motion describing the relation among the forces in the free-body diagram is: $m\dot{v} + bv = u$

This is a first-order ordinary differential (ODE) equation with the velocity *v* as the unknown function. As studied in your calculus courses, many methods exist to solve this ODE, e.g., separable variables or the Ansatz method. With either method, the solution is given by (verify it!)

$$v(t) = e^{-\frac{b}{m}t}v(0) + \frac{1}{m}\int_0^t e^{-\frac{b}{m}(t-\tau)}u(\tau)d\tau$$

Ok, nice. Is there any relation between this solution with complex numbers? One can also solve the equation using the Laplace transform because it allows one to transform the problem of solving an ODE into the problem of solving an *algebraic equation*, which is easy. The Laplace transform of the velocity is

$$\mathcal{L}v(t) = V(s) = \int_0^\infty e^{-st}v(t)dt$$

where $s \in \mathbb{C}$ is a complex variable! For ease of exposure, suppose the input torque

$u = 0$. Using a Laplace transform table (check any book to verify it), let us apply the Laplace transform operator at both-hand sides of the velocity ODE above (second equation of this page)

$$(ms + b)V(s) = v(0)/m$$

After some simple algebraic manipulations, this results in the following expression:

$$V(s) = \frac{1}{s + \frac{b}{m}} v(0)$$

Using the inverse Laplace transform, one can easily verify that the resulting time domain function $v(t)$ is the same for the solution equation (the last equation on the previous page) with $u = 0$. Notice that the denominator of $V(s)$ is zero when the complex variable is $s = -b/m$, which is nothing but the rate of the exponential function in the solution of the velocity ODE. If for whatever the reason b becomes negative, the exponential in the solution will blow up, and so the velocity; what would happen to the car in this case? Thus, by just solving the equation $s + b/m = 0$, one can predict the behavior of the velocity over time without necessarily solving the whole ODE. Interesting, isn't it?

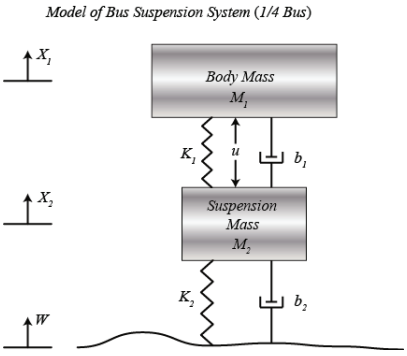

Figure 2: Suspension system (Suspension: System Modeling, n.d.).

The key message of this first part is that using "*complex*" variables (in this case, with an imaginary part equal to zero) can help to simplify some physical modeling and analysis problems.

Complex numbers in analyzing a Car's suspension system

Continuing with analyzing the car shown in Figure 1, we now consider the problem of analyzing the suspension system whose schematic representation is shown in Figure 2.

Let us consider only the suspension mass M_2 and assume that all the forces from the interaction with the body mass M_1 are arranged in a resulting force F_1, and the terrain position is plat ($W = 0$). The free-body diagram (left for the reader as an exercise) and Newton's second law yield the following force balance equation of motion: $M_2\ddot{X}_2 = F_1 - K_2X_2 - b_2\dot{X}_2$

Where K_2 is the spring constant and b_2 the Coulomb friction coefficient. This equation of motion is a second-order ODE where the time position function $X_2(t)$ is unknown. There are also several explicit methods to solve this equation, but

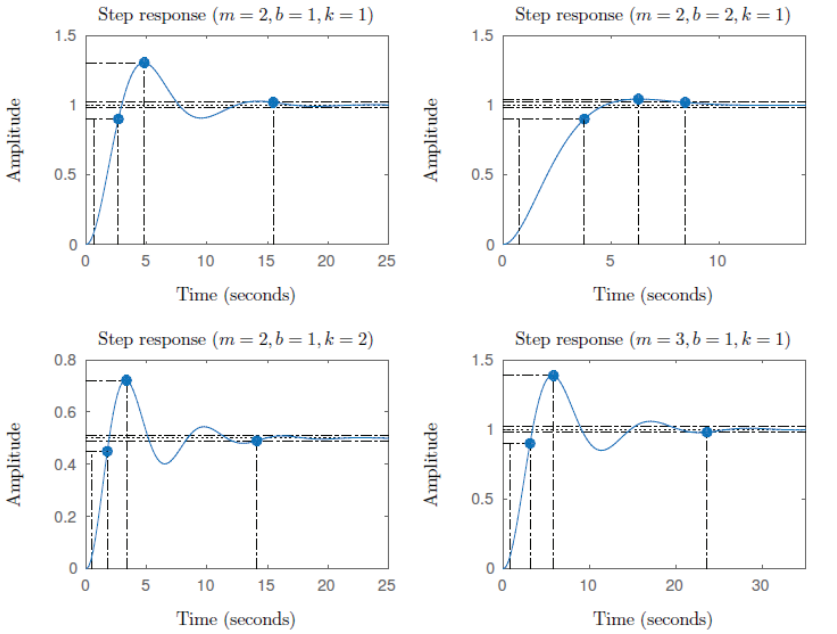

Figure 3: Responses of the system for different values of m, b, k (Reyes-Baez, 2022)

extrapolating the previous analysis for the first order ODE, let us apply the Laplace transform operator to both hand-sides of the second order ODE (the last mentioned equation) with zero initial conditions (i.e., $X_2(0) = X_2(0)$), and solve for X_2. Then, one gets the algebraic equation of degree two given by:

$$(M_2 s^2 + b_2 s + K_2) X_2(s) = F_1(s).$$

Notice that the second-order ODE and the degree-two algebraic equation have similar structure, but the first is differential and the later is algebraic! After some computations, one can solve the algebraic equation for $X_2(s)$ as:

$$X_2(s) = \frac{\frac{1}{M_2}}{s^2 + \frac{b_2}{M_2} s + \frac{K_2}{M_2}} F_1(s)$$

Without going into details, one can easily show that the roots of the denominator in the previous expression for $X_2(s)$ are related to the exponential rates of the solution in the time domain $X_2(t)$. Hence, the solutions of the equation $s^2 + b_2/M_2 s + K_2/M_2 s = 0$ are key for understanding the behavior of the suspension mass position

over time. Let us rewrite this equation as follows:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

With $\omega_n = \sqrt{K_2/M_2}$ the natural frequency of the system and $\zeta = b_2/2\sqrt{K_2M_2}$ the damping ratio of the system. By the fundamental theorem of algebra, this equation has two solutions given

$$s_{1,2} = -\omega_n\zeta \pm j\omega_n\sqrt{1-\zeta^2}$$

Clearly, these solutions are complex conjugate numbers and are parametrized by the natural frequency w_n and damping ratio ζ. The real part of both solutions is $\mathrm{Re}\{s_{1,2}\} = -w_n\zeta$, and the imaginary part is $\mathrm{Im}\{s_{1,2}\} = \pm\omega_n\sqrt{1-\zeta^2}$. Let us play with the mass, spring constant, and friction coefficient values and consider the following numerical values $M_2 = m$, $b_2 = b$, $K_2 = K$ as in Figure 3.

One can appreciate that the friction coefficient is modified from the top-left figure to the top-right, with an effect of "*damping*" the oscillation that the first plot initially has. This is because the friction coefficient $b_2 = b$ is directly proportional to the damping ratio ($\zeta = b_2/2\sqrt{K_2M_2}$), and at the same time, it modifies the imaginary part of $s_{1,2}$ (see the last formula on the previous page).

Likewise, the spring constant is modified from the top-left plot to the bottom-left one. It is appreciated that the plot response does not reach the value of 1 and has more oscillations. Recalling the expression of the natural frequency ($\omega_n = \sqrt{K_2/M_2}$), one can see that increasing the stiffness of the spring also increases the natural frequency of the system. In consequence, the response plot has more oscillations in the transient time. Changing the stiffness also modifies the real and imaginary parts of the complex numbers

$$s_{1,2} = -\omega_n\zeta \pm j\omega_n\sqrt{1-\zeta^2}$$

Finally, to go from the top-left plot to the bottom-right one, the mass $M_2 = m$ is changed from 2 to 3. This decreases the value of the natural frequency $\omega_n = \sqrt{K_2/M_2}$ and the damping ratio $\zeta = b_2/2\sqrt{K_2M_2}$ making the plot take longer to reach a constant value. At the same time, it induces a more oscillatory behavior of the time response.

Therefore, the good performance of the suspension system will depend upon the values of the mass, coefficient friction, and spring stiffness. The design of this type of engineered system should work well under uncertainty and disturbances. The appropriate selection of the parameters is equivalent to finding good locations of the complex numbers

$$s_{1,2} = -\omega_n\zeta \pm j\omega_n\sqrt{1-\zeta^2}$$

As designers of engineering systems, one must ensure the natural frequency and damping ratio induce a decent bandwidth concerning the type of disturbances with which the system will interact. In particular, to avoid disturbances with frequencies close to the system's natural frequency because the undesired resonance behavior

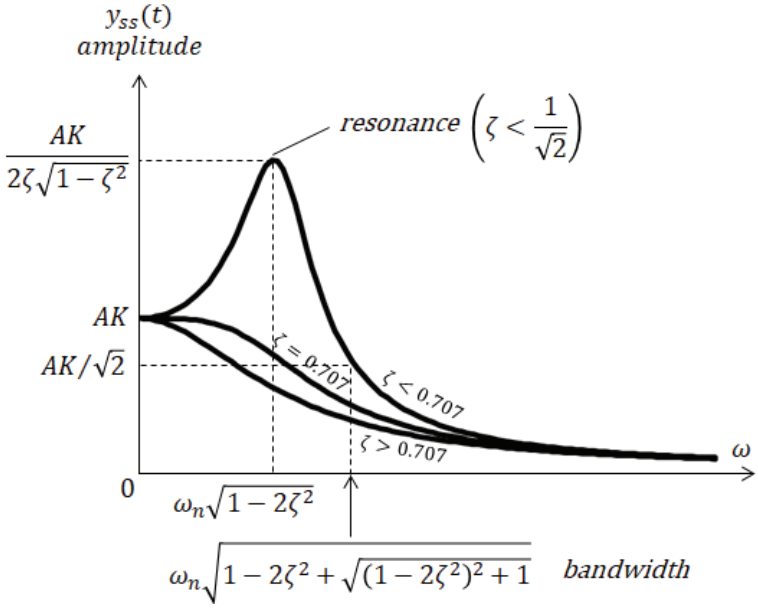

Figure 4: Magnitude frequency response in relation to the natural frequency and damping w_n ratio ζ

may be present. The following figure shows the relation of w_n and ζ with the system's frequency response $X_2(s)$.

Conclusions

In this brief document, I tried to sketch the importance of complex numbers in the dynamic analysis of physical systems. One can perform deep analysis using complex numbers to understand a physical system's time domain and frequency behavior. This is very important in practice as this type of analysis is necessary before building up the systems. This is the focus of the course Control Engineering.

References

1. Control Tutorials. (n.d.). Control Tutorials for MATLAB and Simulink - Home. Retrieved March 8, 2023, from https://ctms.engin.umich.edu/CTMS/index.php?aux=Home

2. Cruise Control: System Modeling. (n.d.). Control Tutorials for MATLAB and Simulink. Retrieved March 8, 2023, from https://ctms.engin.umich.edu/CTMS/index.php?example=CruiseControl&section=SystemModeling Nise, N. S. (2019). Control Systems Engineering (8th ed.). Wiley.

3. Reyes-Baez, R. (2022). Lecture Notes of Control Engineering. In Tutorial 3 (3). University of Groningen.

4. Suspension: System Modeling. (n.d.). Control Tutorials for MATLAB and Simulink. Retrieved March 9, 2023, from https://ctms.engin.umich.edu/CTMS/index.php?example=Suspension&section=SystemModeling

Horoscope

By Hester Braaksma

Feeling lost and in need of some guidance? Let yourself be guided by the position of the stars, just as Galileo Galilei did in his time. I got in touch with a higher force, better known as Chat GPT, and will help you make important decisions being thrown at you.

Aries (March 21 - April 19)
You're feeling full of energy and eager to take on new challenges this week. Skip the drinking and procrastinating for once and focus on getting your shit together instead. Your natural leadership skills will be in high demand, so take the initiative and don't be afraid to break new ground. This is a great time to pursue new opportunities, especially in your career. However, be careful not to burn yourself out, and make sure to set aside some time to play a few hands of klaverjas in the room.

Taurus (April 20 - May 20)
Trust your instincts and don't be swayed by the opinions of others this week. Your determination and focus will bring success in both your personal and professional life. Take time to reflect on your priorities, and focus on what truly matters to you. be mindful of how excessive drinking can jeopardize your academic and financial goals. Remember that it's okay to say no and prioritize your well-being. Consider exploring new non-alcoholic drinks and connecting with friends who support your personal and academic growth.

Gemini (May 21 - June 20)
As a Gemini, you love to socialize and connect with others. However, be aware that excessive drinking can negatively impact your relationships and academic perfor-

mance. This week, try to find a balance between having fun and being responsible. Consider exploring new social scenes , for example joining an FMF or Sirius A activity. Or choose to go into the university library instead of 'studying' at Francken.

Cancer (June 21 - July 22)
Embrace your emotional side and trust your gut feelings this week. Your intuition is strong, and it will guide you in making important decisions. Take time to reflect on your emotions and what truly matters to you, and don't be afraid to express your feelings. Send a nice message to your parents or other family, like the Francken board, to claim good luck. This is also a good time to focus on your well-being and take care of your physical, emotional, and mental health. Switch for example that vlamtosti for a vegetarian one!

Leo (July 23 - August 22)
Your confidence and charisma will shine. Be bold in expressing your opinions and ideas, and don't be afraid to take risks. Channel your inner Shia Labeouf and kiss that person, sign up for that event/course or say yes to a night of spontaneous drinking.However, be mindful of not being too self-absorbed and taking others' feelings into consideration. This can be done by accepting all the invitations of chugging a beer together by your fellow students.

Virgo (August 23 - September 22)
This month, Virgo, you may feel a strong desire to organize and plan out your life. Use this energy to your advantage and set some goals for the future. Stay organized and disciplined, and don't be afraid to ask for help when needed. Don't be afraid to take charge of your social life and suggest fun activities for you and your Francken friends to enjoy.

Libra (September 23 - October 22)
As a Libra, you value harmony and balance. This month, focus on maintaining balance between your social life and academic/work obligations. Don't forget to take some time for yourself as well. Try out some new beer varieties and indulge in a game of klaverjas to unwind. Your relationships with loved ones and colleagues will be strengthened, and you'll feel a sense of balance and harmony. However, be mindful of not being too passive and avoiding confrontation.

Scorpio (October 23 - November 21)
This month is a good time to focus on your personal growth and self-improvement, and to make changes that will bring you closer to your goals. You may find yourself drawn to Franckenborrels that involve drinking. While it's important to have fun and let loose, make sure you're not putting your grades or reputation at risk. Remember to stay hydrated and avoid overindulging. Keep an eye out for potential drama with friends who may not have your best interests at heart.

Sagittarius (November 22 - December 21)
Your adventurous spirit will lead you on new journeys and experiences this week. Stay open-minded and embrace new perspectives, and don't be afraid to step outside your comfort zone. However, when it comes to drinking, it's important to stay safe and responsible. Try to find a balance between having fun and taking care of yourself. Consider trying new (alcoholic) drinks and say yes to crazy ideas.

Capricorn (December 22 - January 19)
Focus on your goals and stay disciplined in your approach. Your hard work and determination will pay off, and you'll make steady progress towards achieving your ambitions. As a hardworking and disciplined Capricorn, you may find it difficult to let loose and have fun. However, allow yourself to relax and enjoy some socializing with your fellow Franckenmembers.

Aquarius (January 20 - February 18)
Your innovative and independent spirit will be in high demand this week. Thinking outside the box will help you during exams and offer unique solutions to problems. Your creative abilities will also be heightened, and you'll find new outlets for self-expression. However, be mindful of not being too detached or unapproachable, and make sure to stay connected with those around you and try to join as many Francken activities as possible.

Pisces (February 19 - March 20)
Your compassionate and empathetic nature will be highlighted this week. Take time to understand the perspectives and feelings of others, and use your intuition to guide you in making decisions. You are extra susceptible to peer pressure the coming month, so try to focus on making your own decisions instead of ending in the Negende Cirkel every day.

Francken Abroad

By Kathinka Frieswijk

New Haven, the city in which Yale University resides, is renowned for its beautiful buildings, Pepe's pizza, and numerous gun violence incidents. One of its main gems is the stunning campus of Yale. Picture Hogwarts, but instead of a lake, it is located next to a lawn. At this square of grass (called the Green), many homeless congregate and lead awareness campaigns, publicly devoting their lives to promoting awareness of the existence of drugs. This peculiar duality of preppy college life on the one hand and derelict living circumstances on the other hand makes New Haven an interesting city.

For the people who are not familiar with me, I was chair of Francken in the year 2017-2018. But even before that, I spent so much time in the Francken room that

Figure 1: Californa (hiking near Standford)

Figure 2: Yale University campus

I was practically furniture. Currently, I am working as a PhD student in the engineering department, and last August, I left Groningen to conduct research at Yale for a few months. Doing research at Yale University was an amazing experience. My host Professor A. Stephen Morse is an exceptional man, both in personality and academic accomplishments. As is tradition, the engineering building at Yale is one of the most outdated and ugly buildings. When the temperatures dropped outside, I had to wear a coat and gloves in the office to keep myself warm, but I did have an office with a magical view of Hogwarts building zás and was very much at peace working there.

Without a car, grocery shopping is quite the expedition in US cities like New Haven, as the affordable supermarkets are often far from the city centre. Once a week, I would take the bus to Walmart and buy food for a week. On its way to Walmart, the bus passes through Fair Haven, one of the riskier areas. As it is inconvenient to live in the US without a car, the bus to Walmart is mainly occupied by people of little means, e.g. the homeless (the bus was free and warm) or residents of Fair Haven. To prevent myself from becoming a target in the more dangerous parts of New Haven, I'd wear run-down sports leggings, because I hypothesised that I was less likely to be targeted if I looked like I did not have enough money to buy sports leggings without holes at Walmart. Whether this was effective or not I

cannot tell, but I can luckily declare that I haven't experienced a single robbery attempt. A colleague at Yale, however, mentioned that it is common to carry around a 'safety 20'. When someone decides to rob you, you quickly hand them 20 dollars so they will leave you alone, as this is usually around what they need to get their drug fix.
Despite New Haven's safety concerns, there are parts of the city that I love. I already mentioned the campus of Yale, but I also loved the streets of East Rock with its beautiful houses. From the clifftops in East Rock Park, you have an overlook of the entire city, and you can even spot Long Island (New York State) on the other side of the water. In October, Steve helped me make arrangements to visit Professor Ji Liu at Stony Brook University on Long Island, and I had a lovely time there. Stony Brook has a different atmosphere, as the university is in a more remote area, has a lot of trees on campus, and even the occasional deer that is not startled by the presence of humans (as no one hunts deer there).

Figure 3: Yale University - Building zás

Growing up watching Gilmore Girls (based in Connecticut), it was a wonderful experience staying on the East Coast during autumn, soaking up the Gilmore atmosphere while watching the trees display an impressive array of vibrant colours. Whereas autumns in the Netherlands are grey and bleary, the autumns in Connecticut are colourful and uplifting. Climate change appears more prominent in the US than in the Netherlands, however, causing weather oddities (I wouldn't be Dutch if I neglected to discuss the weather). I remember this particularly hot day in November when I went for lunch at one of the residential colleges and noticed semi-nude students sunbathing on the courtyard grass.

After I visited Yale, I went to Palo Alto in California, and there, the weather was se-

Figure 5: Yale University College football game

verely abnormal too. As I made my way to the West Coast, the Sunny State tried to make me feel welcome by imitating the climate of my home country. Due to the heavy rainfall, California residents even started singing the "songs of my people" since weather complaints resonated throughout the area for every increase in flood risk. Because of the nasty weather, I spent my time hibernating in different Google buildings, enjoying the free food, iced vanilla lattes, and massage chairs. My other half Santiago was doing an internship at Google X in Mountain View and was able to grant me access. After many days, the weather finally cleared up, so we could go hiking in the beautiful hills surrounding Mountain View (we coined them 'City View'). High points are the 1800-year-old sequoia tree, the random coyote cruising by, and the top of Mount Everest (8.849 km).

Finally, after almost half a year in the US, it was time to return to the Netherlands. The flight home was the most comfortable flight I have ever had. Since there weren't many passengers, I slept through most of the flight by lying horizontal while occupying four chairs. It is an odd reality that while the planet is heating up, there are still planes flying (mostly) empty. Are we even trying?

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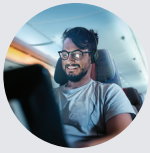

AEROSPACE

GROUND TRANSPORTATION

SPACE

"The traineeship gave me the opportunity to quickly and broadly orientate myself within the organization. Together with the other trainees we formed a close team. After my traineeship, I started working at the sales department."

Kevin
Former Trainee

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Expedition Strategy

By Rick Geling

If you were around during the first semester, you might remember Sjoerd and I promoting an event called Expedition Strategy. Our promotion turned out to be quite successful, but if you didn't sign up, you might be wondering what it's all about. As the current treasurer of the Expedition Strategy board, I'd like to take this opportunity to tell you about our nice event before I hand over my duties.

Expedition Strategy was founded in 2011 by a group of University of Groningen students. The aim of the event is to give students a glimpse into the high-end and

competitive world of strategy consulting. To do so, an inhouse tour is organized at the offices of leading strategy consulting firms. To simulate this competitive environment, top students from the VESTING, TBV Lugus, FSG, FMF, and TFV 'Professor Francken' associations are selected by recruiters from participating companies.

Before we dive further into the event, let's take a moment to introduce the world of strategy consulting. At its core, strategy consulting involves providing advice and recommendations to the highest levels within a company. It's a field that requires a diverse set of skills, including strong analytical abilities, people skills, time management, and flexibility.

Now that we have a better understanding of what strategy consulting involves, let's take a closer look at our event by reflecting on the most recent edition held in November. The first step in the process was for participating companies to collectively select students from the pool of applicants. While the selection process can be competitive, please don't be afraid to sign up. Your CV often has more to offer than you think, especially if you have a strong background in math and physics. It's not without a reason we had a lot of Francken participants this year ;).

After the selection process, we kicked off the event with a "borrel" to bring all the participants together and allow them to get to know each other. This was followed two days later by a case training session with KPMG. Both events were held in Groningen, providing participants with a chance to become familiar with case studies and strategy consulting in general.

One week later, the main program for Expedition Strategy kicked off. We arrived

in Amsterdam on Sunday and enjoyed a lovely dinner together to start things off. The next day, we visited McKinsey & Company which would be the start of a busy week full of office visits. Throughout the week, we got to see the following companies: BCG, OC&C, Roland Berger, Bain & Company, and Strategy&. Each visit had a mix of presentations, case studies, dinners and borrels, which gave everyone a chance to learn more about the firms but also to meet lots of people.

Possibly the most memorable highlight of the week was the party bus that took us from OC&C's office in Rotterdam back to Amsterdam for a night out. This led to some sleepy eyes during the morning program, but luckily, the companies had some really good coffee to perk everyone back up (even better than the coffee in the Francken room).

As my time as the treasurer of the Expedition Strategy board comes to an end, I'd like to end by recapping my year. Although the registration for the board just closed,

I thought it would be nice to share a bit about what it was like. I had a fantastic time organizing the event with a diverse group of students with various backgrounds. It was a great opportunity to gain more experience in organizing, promoting, budgeting, and more. Plus, I got to interact with students from different studies both during the event and prior to the event. And let's not forget, I also got to participate in the event itself ;).

In conclusion, I hope this piece has given you a better understanding of both the event and the experience of being on the board. If you're interested in participating in the future, keep an eye out for announcements and don't hesitate to apply. It's a valuable opportunity to develop new skills, make meaningful connections, and have fun while doing it!

Bob's Adventures

By Anna Kenbeek

Bob joined me for about a week and we became close friends: We discovered that we both like karaoke, that the letters in his name were all the next letters in the alphabet with respect to the letters in my name, and went on indoor and outdoor adventures. I introduced him to my friends, colleagues and sports buddies, and many of them loved him, although some thought he was a bit weird (probably because he was naked).

The week started off with a fancy dinner, where a great amount of 38 old board members of Francken gathered to provide Jeff de Hosson with some good company. And great company it was! Almost all attendees dressed with jackets and ties and we got some inspiration for Bob's outfit. I brought him home safe after the dinner

before I went back into the city to attend a party at Dizkartes, dressed as a jellyfish made from a thrifted pink mosquito net. As Francken board members tend to be really persistent when going out, I could join them again after the sea-themed party.

On Monday we were still a bit hungover from Saturday and Bob took a day off. In the evening, he joined my second time doing weekly groceries, and we felt very civil and organized. The day after, I brought

Bob along to the Avantium headquarters in Amsterdam, where I spent most of my workdays. Unfortunately, we didn't have puppy sized lab coats, so he didn't get to see the labs, but he did get a taste of the atmosphere and lunch. To satisfy his curiosity, after work we went to bouldering hall 'Het

Lab' just around the corner with two colleagues, where we tried some challenging routes and learned that Bob was scared of heights. On Wednesday evening he came along better, when we went ice skating and he joined for quite a few rounds. Carrying a puppy makes a good balance exercise, as it forces you to keep your arms still.
Bob and I joined karaoke on Friday with a group of colleagues. We had a private karaoke room in a bar in Amsterdam, to protect other visitors' ears. In the second karaoke bar, we met some actual good singers, much more comfortable for Bob's young puppy ears. Joris joined as well and we had a great time, maybe danced on the table, and later went to another bar to enjoy some good live rock. I got inspired to make him a waterproof beercoat.

The next day, Chantal (former seceratary of Francken) came to visit and we made a cycle trip through the dunes, interrupted by sandy obstacles. We drank beer on a terrace where a cute sparrow family stole the crumbles of our bitterballen (they loved sweet chili sauce!). In the evening, we played a funny game called Throw Throw Burrito. On Sunday we travelled to Eindhoven where we (a puppy and a pyramid) celebrated carnaval with three friends from high school (Assepoester, a penguin and a stylish goldfish). On Monday we had a day off to recover and went shopping at a fabric shop in Amsterdam. I had no idea where to start making a pattern for his outfit, but finally applied my very basic casting skills

to make a model out of unbleached cotton, before I cut the Francken sweater and sweatpants from the bought fabric. Hopefully Bob feels more comfortable in his new casual and cozy outfit during his upcoming adventures!

A little preview of Bob's outfit

For the bottom-up design of the future...

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We participate in programs of Bachelor and Master levels in the field of Physics and Chemistry. But, since it is our mission to train a new generation of researchers in cross-disciplinary approaches to research and equip them with the diverse skills required by modern science, we have initialized programs breaking the traditional boundaries between disciplines like e.g. the interdisciplinary Top Master program Nanoscience and the High Tech Systems and Materials Honours Master, which tackles real-life product development challenges in the same interdisciplinary fashion.

Additionally to the Bachelor and Master education, the Zernike Institute has the responsibility to train almost 200 PhD students to become independent, high level scientists. The main component is 'hands-on training', working side-by-side with the research staff of the institute.

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Member's input

By Rebecca Bult

Hello, I am Rebecca and I guess you can call me a presjaars. This year I started studying physics and quickly joined Francken even though the association is for applied physics students. After I started learning to klaverjas, Francken quickly became like a second home for me. Every day before, after, and during class, I was at Francken, maybe a little bit too much for my study's sake. There is always someone to talk to or play klaverjas with. I tried to study in the Francken room but that's basically a lost cause. It's actually amazing how many people come to Francken who think they can get some work done there, find out they have done nothing, and still come back the next day to try again.

Each committee always organizes fun activities and events. It's just hard to not want to be at all of them. Even more when an older year student or a board member (lightly) pressures me into coming. Just make sure you have a spine and a good plan on how and at what time you want to go home before you go to one of the borrels. Otherwise, there is a big chance you'll end up in de negende cirkel till 4:00 (this happened too many times to me). But Francken is not only good for partying and having fun. It's also a place where I made a lot of friends. Not only from the same academic year but also from other years. It's sometimes very nice to also have older students tell you how life is going to look like the longer you

study. I remember when it was getting clear that I was in trouble with my BSA, how a lot of older students wanted to help me. They tried to support me and whenever I had a question there was always someone who wanted to explain the problem to me. There were even people who tried to teach the whole course in 10 minutes but that didn't work sadly. So even though Francken can be a bit too much fun it can also help a lot. And even though I needed to de-enroll from my first year, I'm still in the Francken room almost daily. And next year I will be a sjaars again. So I hope to see everyone in the Francken room.

Comic

By Julia Aizpún

Koke met Sjoke: Imaginary risotto

By Sjoukje de Jong

This recipe is perfect for when you want the idea of risotto, but not risotto. Eating this meal you can almost imagine eating risotto without having to wait so long for the risotto to get cooked. This recipe is also perfect for someone that does not like dishes but does own a big wok pan or something similar. Fun fact: when I first tried out this recipe, my parents' dog was giving birth in a room next to the kitchen. I can tell you from experience that checking how many puppies there are is a great way to fill time when waiting for the ingredients to cook!

Ingredients for 3 (or 2 hungry) people:

- 300g chicken breast
- 1 teaspoon oregano
- 1 onion
- 2 cloves of garlic
- 150g orzo
- ½ tablespoon lemon juice
- 200ml cream
- 30g parmesan cheese
- 150g spinach
- 50g sun dried tomatoes (cut in thin slices)
- Olive oil, pepper and salt

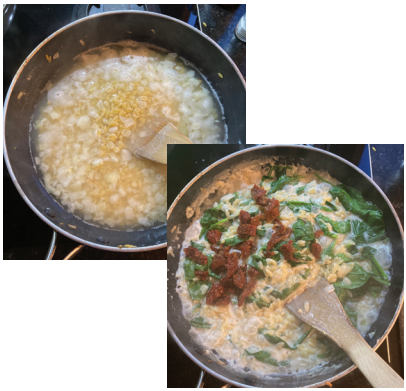
To veganise this recipe, replace chicken breasts with vegan chicken, cream with coconut milk and leave out the parmesan cheese.

Now to assemble this fantasy:

1. Heat some olive oil in the pan and cook the chicken, add oregano, salt and pepper to taste.

2. When the chicken is cooked (not pink on the inside), take it out of the pan.
3. Again, heat some olive oil in the pan and fry the onion and add the garlic a few minutes after you added the onion. Finally add the orzo and fry everything for a bit.

4. Add 500ml of water to the pan together with the lemon juice and bring it to a boil. Add the cream, parmesan cheese, spinach and sun dried tomatoes. Combine it until the spinach has shrunken.



5. Add the chicken back to the pan and heat the complete thing until the orzo is cooked (taste to check). Add some pepper and salt to taste and enjoy this delicious meal!

I hope you enjoy this meal, me and my family definitely did!

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Prof Dr. Marcos Guimaraes
Assistant Professor at the Zernike Institute for Advanced Materials of the University of Groningen

SPEAKERS

Quantum Computing for Everybody
Mr. Richard Versluis
QuTech Systems Engineer

Transducer Devices: The Backbone of the Future Quantum Internet
Dr. Thierry van Thiel
QphoX Senior Quantum Engineer

Professor in Physics of Quantum Devices
Dr. Caspar van der Wal
Physics of Quantum Devices, University of Groningen

Eindhoven University of Technology
Full-scale Quantum Computing: The Quantum Co-processor

Marijn Venderbosch & Robert de Keijzer

The Francken Dread

By Hannelys Posthumus

Yet another day in the Francken room
The smell of tosti's and sweet perfume
One group playing klaverjassen while others playing chess
I have to confess, sometimes life is a mess
But Francken is a bless

Yet another night at a Francken event
Yellow gold fluid in glasses is present
There is never too much escalation
Neither a too high alcohol concentration

Yet another morning hungover in my bed
To Francken I fled
To continue the dread

Shell interview

By Siem Kuijpers

Together with Wout Trox and the Commissioner of External Relations of Francken Sjoerd Buitjes, we went to the NAM (Nederlandse Aardolie Maatschappij, which Shell and Esso founded) in Assen to have an interview with Simeon Molenaar who was at that time a Concept Engineer for Energy Transition of Shell. He also studied at the RUG doing a bachelor's and a master's in industrial engineering. Simeon did his internship at the NAM in Assen and later his graduate internship at Shell in Amsterdam.

As a process engineer, he and his team made a manned installation unmanned, lowering the number of people needed offshore and therefore reducing the cost and making it safer. In his career, he has always been involved with the energy transition. At the 70th year anniversary of NAM, they put out a price question for energy storage. His main focus is on blue Hydrogen, blue Hydrogen is made in the same way as grey Hydrogen from natural gas where CH4 is converted to CO2 and H2, but the difference lies in the fact that the CO2 is captured instead of released into the atmosphere. In the early days of Hydrogen production, CO2 got released into the atmosphere. However, it is quite easy to capture the CO2 since it is released in the production cycle under high pressure and in high concentrations. They can even put the captured CO2 back into the ground where they initially took the natural gas from.

These gas sources have multiple layers which prevent the gas from escaping, in the

Netherlands these most of the time consist of salt layers through which the gas can't escape. When they are finished with the gas extraction they seal off these multiple layers. So when they store the CO2 back in those gas sources once they're filled up they can close them again so the CO2 is stored underground.

Shell is also testing with green hydrogen, in this method, they use renewable energy from for example windmills to split water into hydrogen and oxygen by electrolysis. The windmill parks that are built today are still relatively close to the shore, so then you would still use cables to transport the energy to our infrastructure. But in the future when we start building wind turbines further away it might become beneficial to turn the energy into hydrogen before transporting it to the shore. One of their hydrogen pipelines could for example transport about 12 gigawatts, whereas a cable usually caps at around 2 to 4 gigawatts. Another benefit is that the pipelines are about 10 times cheaper compared to the cables. Qbuzz and Shell already have a collaboration where Qbuzz busses in Groningen are driving on green-certified hydrogen.

Now one might ask why is Shell an oil company investing so much money into renewable energy. But today Shell is technically not even an oil company anymore but rather a gas company since they produce more gas than oil. In the end, Shell is an energy company, that operates over the entire chain. From extracting the energy to directly selling the energy to consumers. Shell for example also has the largest LNG fleet in the world. So Shell has a unique position when it comes to developing new energy structures. Building an offshore wind farm for example is not even that difficult, but fitting the wind farm into an energy system is where the difficulties start to arise. So just like in the past, when Shell choose to invest more in gas instead of oil because gas would play a bigger role in the energy transition, Shell also now sees that we would have to leave fossil fuels behind and transition to renewable energies like hydrogen. Therefore Shell plans to be 100% CO2 neutral by 2050, there is no company that invests more in the energy transition in the Netherlands than Shell.

We also asked Simeon about one of the more strange things that he experienced working at Shell. He said that it was extraordinary how much responsibility he was able to get at the start of his career. Making him feel he really makes an impact in the company. At the start of most careers one could usually only imagine.

Puzzle

By Rosa de Graaff

Artist M. C. Escher is known for his surreal artworks, including his painting called "Relativity" which depicts an impossible world of staircases. His works often feature patterns, schapes and reflections, and are frequently used as covers for mathematics books and papers. Although he believed he was not mathematically skilled as he did not have an education in maths or physics, he went to math conferences where he met mathematicians. Particularly, he became friends with Roger Penrose, as they both shared a fascination for tessellations: a surface covered with geometrical shapes (tiles) with no overlaps and gaps. Escher's last drawing is a tessellation made from a figure he drew for Penrose, which he called his "little ghost", and was based on a basic shape for tessellations that Penrose had explained to him.

Penrose aimed to create a tessellation that could not be repeated, which was believed among mathematicians to be impossible until the 1950's. However, he came up with two basic shapes, a dart and a kite, with angles all in multiplications of 36 degrees ('Vo) that, when laid out in a specific way, create such an aperiodic tiling. These tilings had some remarkable properties that were of great benefit for not only mathematicians, but also for physicists and astronomers. It has a local 5 fold rotational symmetry (found in nature in quasicrystals like aluminum-manganese alloys), and can continue till infinity. In order to let the audience explore these aperiodic tilings, Penrose and his father (also a mathematician) created puzzles that were published in the *New Scientist* in 1958. In this puzzle section, I encourage you to play with these puzzles

as a good warm-up exercise for understanding how to visualize abstract concepts. After these puzzles I will continue explaining a mathematical approach for the concept of ''infinity'' on a 2D surface, that was used by Escher in his later career.

The puzzles

1. Staircase for lazy people

The first problem in Figure 1 is to go from A to B, mounting the fewest number of steps. If we turn to the right after four steps and then to the left we climb eleven steps before reaching the top. Similarly, by turning to the left after four steps and to the right after four more, eleven steps are climbed. But B can be reached by mounting only ten steps and also descending three on the way. This is done by turning to the right after four steps and then to the right again and then crossing the bridge. The further problem is to find out why this is possible or, rather, why the staircase is impossible.

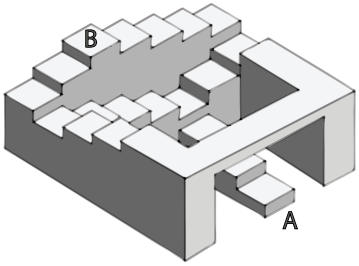


Figure 1

2. Railway mazes

The following type of maze offers a wide scope for possibilities and it does not appear to have been exploited before. Two examples are shown in Figures 3 and 4. The problem in each case is to get from A to B by travelling along the lines but without turning any sharp corners and without stopping and reversing. Thus, in Figure 2, the routes from x to y or from y to z are allowed, but not the one from x to z. The lines may be thought of as representing railway tracks along which an engine with no reverse gear is travelling

Figure 2

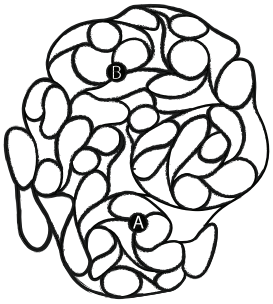


Figure 3

Figure 4

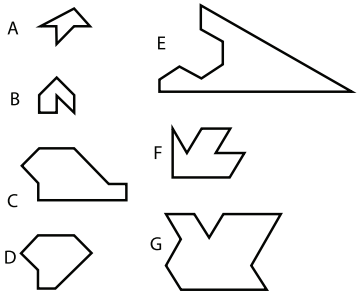


Figure 5

3. Plane tessellations

It is well known that it is possible to cover a plane with an infinite repeating pattern of squares, or of equilateral triangles or regular hexagons. There are, however, many other shapes which can also be built into such a repeating pattern. For example, for the shape A of Figure 5, there is the pattern shown in Figure 6. The reader may care to amuse himself by trying to do the same for the shapes, B, C, D, E, F and G. In Figure 6 the shape A is forced to take up four different orientations, two of which are A turned over, that is, reflected. How many orientations must be used in the corresponding patterns for B, C, D, E, F and G? In each case the solution is unique.

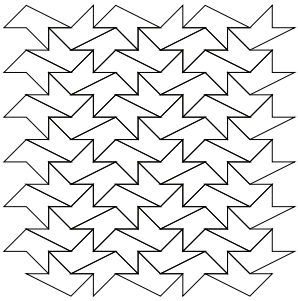


Figure 6

4. Rescuing a kite

Pluis has been flying a kite in a variable wind and it has come down in a tree, as shown in Figure 8 (next page). To retrieve it he must reach the tree on foot carrying his reel of string, of which he has plenty, which is now tucked in his belt. The string must not be tangled, and not be caught behind any of the arches, when he reaches the tree. He can only cross the river by the bridge or the viaduct and he can only reach the ground by walking down the steps. Trace the path Pluis must take in order to rescue his kite. (Rosa's sidenote: I tried to trace the original picture but it might not be clear that in fact you can walk next to both sides of the river underneath the three arches of the bridge).

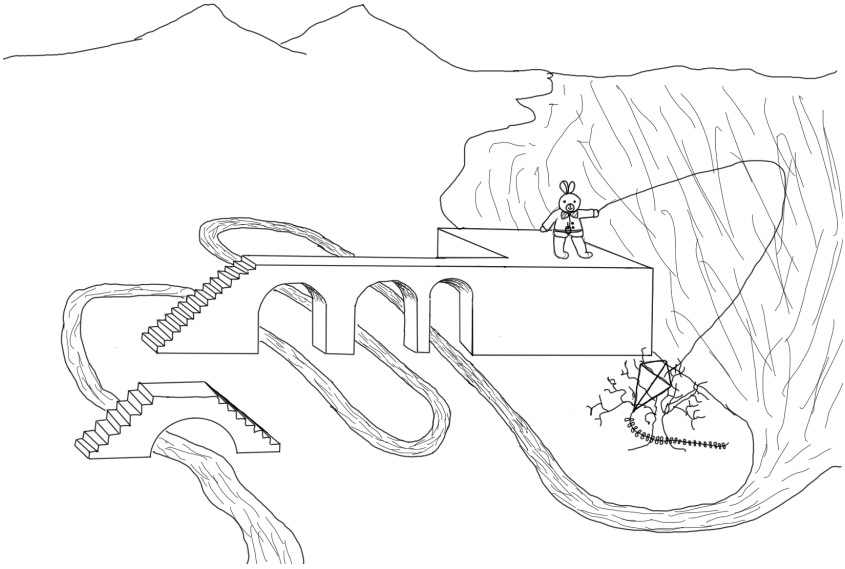


Figure 8

6. The biliard table problem
This is a problem for the mathematically inclined reader. The puzzle is to design a shape for a billiard table with the property that there are two regions, A and B, on the table such that it is impossible to hit a ball from any point inside A to one inside B. The table is to be without pockets or corners, that is to say its shape is a smooth closed curve. It is to be assumed, of course, that the angle made by the ball at the cushion on impact is exactly equal to the corresponding angle made on the rebound, and that there is no friction. Is it possible to extend this result to three dimensions? That is, can we find a smooth closed reflecting surface (mirror) which contains two regions with the property that a source of light placed in one cannot be seen from the other?

Penrose diagram in a flat universe

The goal of Penrose was to show what precisely it means to draw "impossible objects": as you were following the lines of the figure, sudden changes of perspective were needed (think of the problem 'Staircase for lazy people'). This idea can be extended to some "impossible objects" in physics, such as the concept of infinity. For this explana-

tion we need General Relativity, a field in which infinities are well studied by Penrose and for which he created the mathematical tools.

The concept of spacetime is fundamental in General Relativity, it describes 3D space and 1D time in a single 4D union. An example of such a spacetime is the Minkowski space described by the metric:

$$dS^2 = -(cdt)^2 + dx^2 + dy^2 + dz^2$$

where c denotes the speed of light, dt the time interval, dx, dy and dz the interval in spatial distances, and dS the line element which specifies the distance travelled in that interval in any given metric of spacetime. It can be visualized via a spacetime diagram in which time runs along the vertical axis and space along the horizontal axis. Assuming the readers of the FranckenVrij have basic knowledge of Special Relativity, we know that light rays are represented by diagonal lines forming a hourglass shape called a light cone. Every point, P, in spacetime has its own lightcone in which events in the future lie on top, past below, present in the middle and the space outside of this shape is often referred to as "elsewhere". The latter is due to the fact that nothing can travel faster than the speed of light, so points outside the light cone is not connected with point P and is called spacelike separated. Points connected by the speed of light are called null separated and inside the light cone is called timelike separated. Although this diagram is very useful to describe, for instance, the path of light and other objects, it is of no use when we want to describe events in the neighborhood of infinity. To extent the diagram to infinity, you would need to have infinite paper to draw it. Fortunately, Penrose managed to find a mathematical tool that allowed us to actually draw spacetime diagrams, including the point at infinity.

In order to make such a diagram, Penrose applied a conformal mapping to the spacetime metric. A conformal mapping is function that transforms a given curve or plane by locally preserving the angles but not necessarily the lengths. Penrose used the following mapping:

$$u \pm v = \tan^{-1}(x \pm t)$$

where (u,v) are the new rotated coordinates of space and time given by $u = t - x$ and $v = t + x$. This is a clever transformation since the tangent is bound to $-\pi/2$ and $\pi/2$ and not to infinity. The new diagram looks like the square in figure 9, and infinity is located on the sides of the square. In this new diagram we have been able to depict the future timelike infinity at i^+, past timelike infinity i^- and spacelike infinities i^0. The path of ingoing and outgoing light rays follow the lines $\mathcal{I}_\pm$ which are called future and past null infinity (similair to our previous light cone but now a different shape). If we analyze the picture, we see the path of an object in green and the path of a photon in yellow.

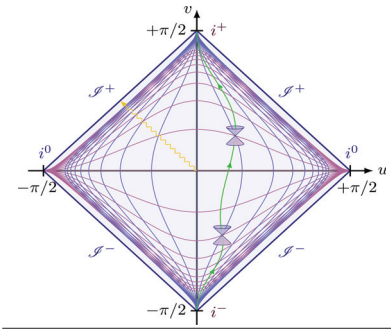


Figure 9: Penrose diagram for flatspace (Neutelings).

Now let's get back to the initial metric we were dealing with, the Minkowski metric. Why go through tedious math if we could just ignore the vague points at infinity? Well, the old metric depicted a flat universe with no heavy objects like stars, black holes or even you. If we include those, we obtain a completely different metric where problems do occur and infinities need to be taken carefully into account.

Penrose diagram for a black hole

When including heavy objects we must also introduce gravity, since Einstein realized that gravity is the curvature of spacetime caused by mass, which can not be described by the simple Minkowski metric. An example of a metric that describes the geometry of spacetime including a heavy object with mass M (for example a star or a black hole) is the Schwarzschild metric,

$$dS^2 = -\left(1-\frac{2GM}{c^2r}\right)(cdt)^2 + \left(1-\frac{2GM}{c^2r}\right)^{-1} dr^2 + r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)$$

where r is the distance from the mass, G the gravitational constant and we use polar coordinates instead of Cartesian. As long as we describe line elements at distances not close to $r = 2GM/c^2$ or $r = 0$, the radius and center of the black hole respectively, we are fine. But once we approach these radii, we encounter singularities. This is problematic when we consider a black hole and we like to know the path a lightray (or matter) would follow while falling into the black hole. The equation will blow up to infinity and does not give us a satisfying physical meaning about the situation. Luckily, we can do the same mapping as before and the solution to that is the Penrose diagram describing (and more importantly, depicting) the spacetime containing a black hole. I will now give the solution right away without the derivation, but note that while doing the derivation extra coordinate transformations are needed before doing the mappinng introduced by Penrose, hence a slightly different notations:

$$\begin{cases} U+V = \tan(u+v) \\ U-V = \tan(u-v) \end{cases}$$

$$\Rightarrow \begin{cases} \cos(2v) = \dfrac{2GM+(r-2GM)\,e^{\frac{r}{2GM}}}{2GM-(r-2GM)\,e^{\frac{r}{2GM}}}\cos(2u) & \\ \sin(2v) = \tanh\left(\dfrac{t}{4GM}\right)\sin(2u) & \text{for } r > 2GM \\ \sin(2v) = \cosh\left(\dfrac{t}{4GM}\right)\sin(2u) & \text{for } r < 2GM \end{cases}$$

These equation might look complicated, but consider v the time axis and u the space axis, see figure 2. The advantage of this dia-

gram is that the most important properties of a black hole are now also included: the singularity and the horizon. These two properties are located at the asymptotic regions where the initial Schwarzschild metric blows up. The horizon, also known as the point of no return, is the radius at which no lightrays are able to escape from the black hole, hence its name. The singularity is the point at $r = 0$ where all the matter and light will fall towards. Lastly, we see still have the different kinds of infinities denoted by $i^{+,0}$

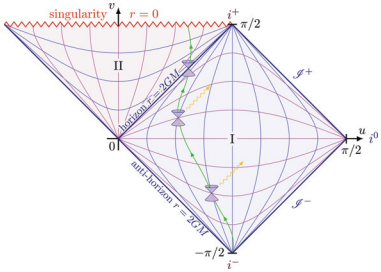


Figure 10. Penrose diagram for Schwarzschild black hole (Neutelings).

and $\mathcal{I}_{\pm}$ just like before.I hope you now appreciate the value of a Penrose diagram. Instead of having to draw
infinity on infinite paper, we have included it at the boundary of the diagram. With equations we can now calculate the path a lightray or object would follow and are able visualize it. The last Penrose diagram including a black hole takes effort to understand and might be a bit confusion. Therefore, I'd like to end this piece with Escher's version of a Penrose diagram including tessellations, see figure 11. It shows the concept of the Penrose diagram where at the edge of the circle the angels and devils are fading away into infinity.

References

1. Erik Kersten. "The Roger Penrose Puzzle". 11 May 2019, posted on the website www.escherinhetpaleis.nl.
2. A. S. Wright. "The origins of Penrose diagrams in Physics, Art, and the Psychology of perception, 1958–62". September 2013, Elsevier.
3. Izaak Neutelings. "Penrose diagrams of Minkowski and Schwarzschild spacetime". Posted on Tikz.net.
4. Anil Zenginoğlu. "How to draw Penrose diagram". Posted on GitHub on September 1st 2022.
5. L.S. Penrose & R. Penrose. "Puzzles for Christmas". The New Scientist, 25 December 1958.
6. J. Hartle. "Gravity - An Introduction to Einstein's General Relativity". 28 januari 2003.
7. Wikipedia for some general information.

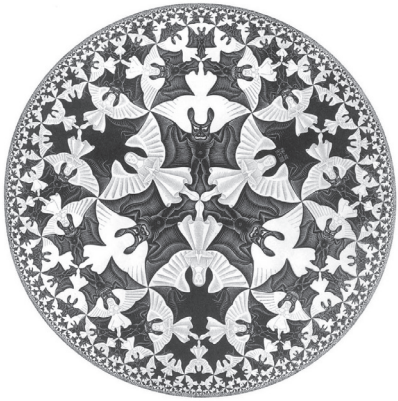
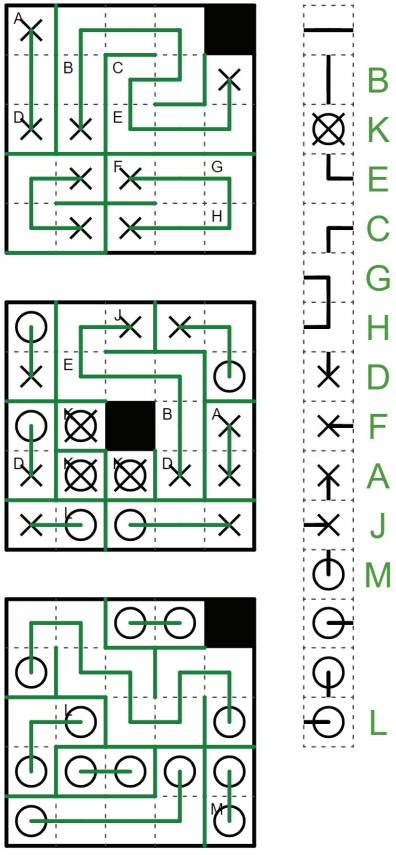
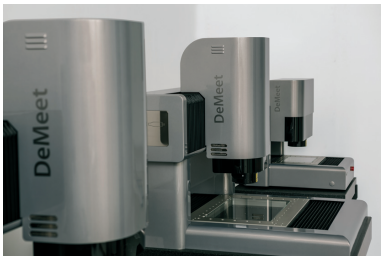


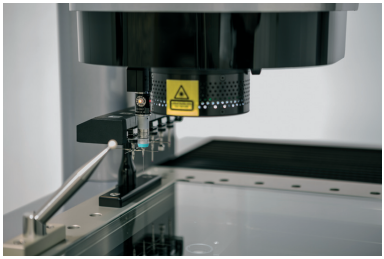
Figure 11. 'Angels and Devils', painted by Escher.

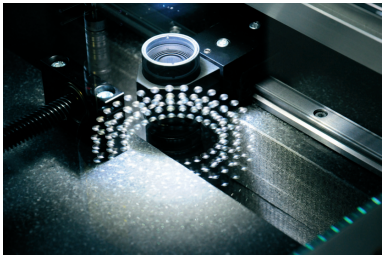
Solution to 27.1

The strange loop solution:









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